

A New Approach For Triangular Intuitionistic Fuzzy Number in Multi-criteria Decision Making Problems

M. Saeed*, M. Khubab Siddique†, M. Ahsan‡, A. Rayees§, G. Rasool ¶

Abstract In this paper, we use triangular intuitionistic fuzzy numbers to solve MCDM problem. we will consider the situation in which data is available in the form of triangular intuitionistic fuzzy numbers. We convert the data in to triangular fuzzy numbers and then by using topsis for fuzzy numbers the ranking is made.

1 Introduction

In our daily life, a decision maker use the data which is not a single value. They have certain merits and demerits for each criteria of alternatives, so data is obtained as intuitionistic fuzzy numbers. In this connection, the decision maker used the different techniques to described decision problems. some times it is not possible for decision makers to make proper decision, their decision is based on the uncertain and imprecise information, so the intuitionistic fuzzy numbers (IFV) can be used to quantify this situation. This situation can be Handel by Triangular intuitionistic fuzzy numbers (TrIFN) as these are more suitable to model uncertain situations. Sqlain et. al [43] gave the application of generalized fuzzy topsis in DM for neutrosophic soft set to predict the the champion of FIFA 2018. Saqlain et. al. [44] worked on the generalization of topsis for neutrosophic hyper soft set using accuracy function. In this paper we use two methods to deal

*Department of Mathematics, University of Management and Technology, Lahore, Email: muhammad.saeed@umt.edu.pk

†Department of Mathematics, University of Management and Technology, Lahore, Email:khubabsiddique@hotmail.com

‡Department of Mathematics, University of Management and Technology, Lahore, Email: ahsan1826@gmail.com

§Department of Mathematics, University of Management and Technology, Lahore, Email: rayeesmalik.ravian@gmail.com

¶Department of Mathematics, University of Management and Technology, Lahore, Email:ghulamqau@gmail.com

with intuitionistic fuzzy numbers. In first method we convert the intuitionistic fuzzy numbers to fuzzy numbers and can use the various fuzzy topsis method which are discussed in [12]. The concept of fuzzy and soft is applied to solve a lot of problems in [48–53]. Saeed et al. [54] explained some basic concepts of the hypersoft

2 Preliminaries

In this section some basic Some basic notion are given.

Let X be a crisp universal set and $I: X \rightarrow [0, 1]$ be a fuzzy set. $m_I(x)$ is known as the degree of membership of x in I , $\forall x \in X$. In 1975, the concept of linguistic variable introduced by Zadeh. A linguistic variable is a variable whose membership function is characterized by word, For example, weather is a linguistic variable whose membership function is low temperature or high temperature, etc.

Definition 1 [4] An intuitionistic hesitant fuzzy sets is a function $X \rightarrow [0, 1]$ defined as $I = \{(x, m_I(x), n_I(x)) | x \in X\}$, where $m_I(x)$ and $n_I(x)$ are membership and non-membership degree of I and $m_I(x), n_I(x) \in [0, 1]$,

Definition 2 [4] For every common intuitionistic fuzzy subset I on X , we have $\pi_I(x) = 1 - m_I(x) - n_I(x)$ called the intuitionistic fuzzy index or hesitancy index of x in I . is the degree of indeterminacy of $x \in X$ to the IFS I . Clearly $0 \leq \pi_I(x) \leq 1$.

Definition 3 [24] Member function for IFS I on the universe of discourse X is defined as $m_I: X \rightarrow [0, 1]$, where each element X is mapped to a value between 0 and 1. The value $m_I(x); x \in X$ is called the membership value or degree of membership

Definition 4 [4] Non-member function for IFS I on the universe of discourse X is defined as $n_I: X \rightarrow [0, 1]$, where each element X is mapped to a value between 0 and 1. The value $n_I(x); x \in X$ is called the non-membership value or degree of non-membership

Definition 5 [1] For a fixed universe X , the IFS I can be described as a function $X \rightarrow [0, 1] \times [0, 1]$ and it can be defined by a pair $\langle m_I, n_I \rangle$ for $x \in X$ $m_I(x)$ denotes the degree of membership of x and $n_I(x)$ denotes the degree of non-membership of x to the set I and $m_I(x)$ and $n_I(x)$ satisfy the condition $m_I(x) + n_I(x) \leq 1$. When $m_I(x) + n_I(x) = 1$, the set I takes the form of fuzzy set.

2.1 Fuzzification of intuitionistic fuzzy numbers

We can use two methods for Fuzzification of IFS from [42], Metod 1 modified as in case of $m_I(X) = n_I(X)$, we have not added hesitancy to any one of $m_I(X)$ and $n_I(X)$ but divide them equally. Here $\varepsilon = n_I(x) = 1 - m_I(x) - n_I(x)$

2.1.1 Method 1

1. If $m_I(X) > n_I(X)$ then change the value of $m_I(X)$ to $1 - n_I(X)$.
2. If $n_I(X) > m_I(X)$ then change the value of $n_I(X)$ to $1 - m_I(X)$
3. If $m_I(X) = n_I(X)$ then add half of hesitancy $\frac{\epsilon}{2}$ to both $m_I(X)$ and $n_I(X)$

2.1.2 Method 2

In this method hesitation is divided in the proportion of $m_I(X)$ and $n_I(X)$.

1. Add $\epsilon \frac{m_I(X)}{m_I(X)+n_I(X)}$ to $m_I(X)$ and $\epsilon \frac{n_I(X)}{m_I(X)+n_I(X)}$ to $n_I(X)$, where

2.2 Defuzzification of intuitionistic fuzzy numbers

The crispification is obtained by mapping $[0, 1] \times [0, 1]$ to R , where R is the set of real numbers is introduced. Here, $X = R$ for *IFSs*. Here IF-defuzzification

function is used to convert membership and non-membership values get crisp values. In this section, formulation and the features of a few defuzzification functions are discussed. Throughout this paper, I represents *IFS*.

1. Intuitionistic fuzzy triangular (*iftridf*) [1] is defined as

$$F(z) = \begin{cases} \leq a, & \text{if } z = 0 \\ \alpha + (b - a)(y + \epsilon) - (\sqrt{m * (c_1 - n)}), & \text{if } 0 < z \leq \alpha_1 \\ (b - a)(y + \epsilon) - (\sqrt{m * (c_1 - n)}) + c - (\sqrt{m * (c_2 - n)}), & \text{if } \alpha_1 \leq z < \alpha_2 \\ \geq c, & \text{if } z = 0 \end{cases}$$

where c_1 and c_2 are arbitrary constants and $z = I(x)$ is the fuzzified value which lies in $[0, 1]$ and is a ϵ small quantity such that $m_I(x) + n_I(x) + \epsilon = 1$ and $0 < \epsilon \leq 1$. and $\alpha_1 = \frac{x-a}{b-a} - \epsilon$ and $\alpha_2 = \frac{c-x}{c-b} - \epsilon$

Note: Hereafter, ϵ is so chosen that $m_I(x) + n_I(x) + \epsilon = 1$ and $0 < \epsilon < 1$.
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2. Intuitionistic fuzzy trapezoidal (*iftradf*) [1] IF-trapezoidal defuzzification function (*iftradf*) is defined as

$$F(z) = \begin{cases} \leq a, & \text{if } z = 0 \\ \alpha + (b - a)(y + \epsilon) - \sqrt{m * (c_1 - n)}, & \text{if } 0 < z \leq \alpha_1 \\ b \leq x \leq c, & \text{if } z = \alpha_2 = 1 - \epsilon \\ (c - d)(y + \epsilon) + d - \sqrt{m * (c_2 - n)}, & \text{if } 1 - \epsilon = \alpha_2 \leq z < \alpha_3 \\ \geq d, & \text{if } z = 0 \end{cases}$$

where c_1 and c_2 are arbitrary constants and $\alpha_1 = \frac{x-a}{c-b} - \epsilon$, $\alpha_2 = 1 - \epsilon$ and $\alpha_3 = \frac{d-x}{d-c} - \epsilon$

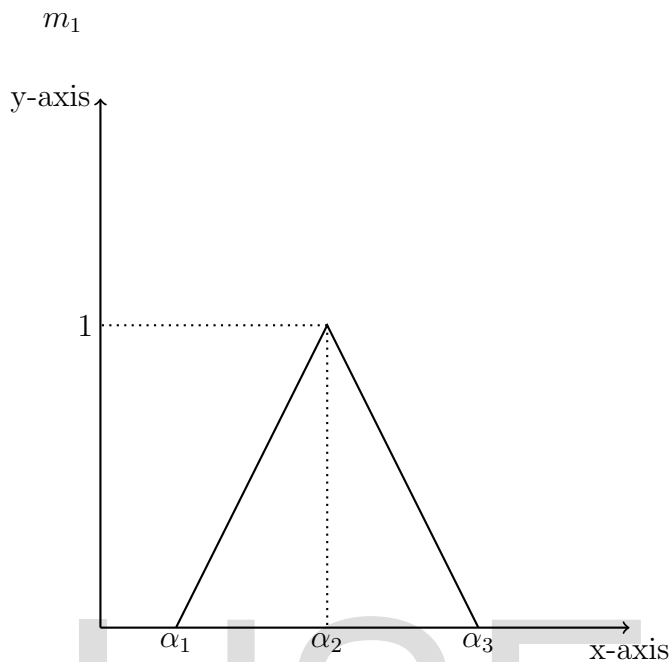


Figure 1: triangular fuzzy number

2.3 Topsis for Intuitionistic fuzzy triangular number

There are many extensions of topsis in literature [[36], [37], [38], [46]] to deal with fuzzy numbers.

2.4 Topsis for intuitionistic fuzzy number

There are many extensions of topsis in literature [[36], [37], [38], [46]] to deal with fuzzy numbers.

One of the most classical and widely-used MADM method is TOPSIS (Technique of Order Preference Similarity to the Ideal Solution) [[22], [33]]. In TOPSIS method positive ideal and negative-ideal solutions considered and distance of each one of the alternatives are compared to those. It has been applied in chain management and logistics, engineering, marketing, and environmental management (for a review, see [[9]]) and found to be very successful.

The TOPSIS is described in the following six steps.

1. Normalize the decision matrix in order to get dimensionless values. The technique vector normalization technique is used as

$$\gamma_{ij} = \frac{x_{ij}}{\sum_{i=1}^m x_{ij}^2}, \quad i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n$$

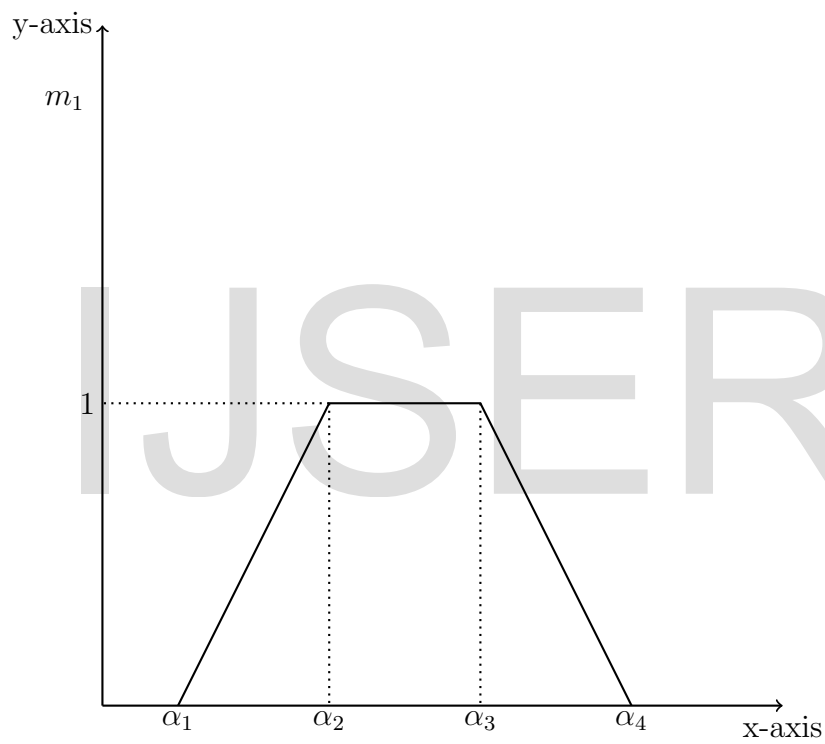


Figure 2: trapezoidal fuzzy number

2. Normalized weighted decision matrix is obtained by multiplying the corresponding values with weight associated with each criteria.

$$v_{ij} = w_j \gamma_{ij}, \quad i = 1, 2, \dots, m \quad \text{and} \quad j = 1, 2, \dots, n$$

3. The positive ideal (A^+) solutions and negative ideal solutions (A^-) solutions are evaluated as follows

$$\begin{aligned} A^+ &= \{v_1^+, v_2^+, \dots, v_n^+\} \\ &= \{(\max_j v_{ij} | j \in B), (\min_j v_{ij} | j \in C)\}, \quad j = 1, 2, \dots, n \end{aligned}$$

and

$$\begin{aligned} A^- &= \{v_1^-, v_2^-, \dots, v_n^-\} \\ &= \{(\min_j v_{ij} | j \in B), (\max_j v_{ij} | j \in C)\}, \quad j = 1, 2, \dots, n \end{aligned}$$

where B and C represents benefit criteria and cost criteria respectively.

4. Calculate the distance from positive ideal solution and negative ideal solution for each alternative.

we can use here many type of distances.

$$D_i^+ = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^+)^2}, \quad i = 1, 2, \dots, m$$

and

$$D_i^- = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^-)^2}, \quad i = 1, 2, \dots, m$$

5. Find the relative closeness C_i of each alternative to ideal solution as

$$C_i = \frac{D_i^-}{D_i^- + D_i^+}, \quad i = 1, 2, \dots, m$$

where $0 < C_i < 1$

6. The alternatives are so Best (ranked that higher relative closeness value C_i) is to worst (lowest relative closeness value C_i).

Various experiments and modifications are made in TOPSIS by changing the normalization process [[10], [39], [40], [45], [47]].

The proper determination of the positive and negative ideal solution [[13], [18]],

Different distance techniques are used for the calculation of the distances from the positive and negative ideal solution [[11], [41]]

3 Proposed Method

The Topsis method is extended to intuitionistic fuzzy environment by using systematic approach. For solving the group

decision-making problem under intuitionistic fuzzy environment this method is very suitable. In this paper, the importance weights of various criteria

and the ratings of qualitative criteria are considered. The proposed method consists of following steps

1. Each decision maker gives the data in form of intuitionistic fuzzy numbers.
2. This data is represented as triangular intuitionistic fuzzy numbers. we will get two ordered tuples for each set alternative and a criteria. One for membership value and other for non-membership value. say as (m_1, m_2, m_3) and (n_1, n_2, n_3) . By using this we will construct Triangular Intuitionistic fuzzy decision matrix and elements in it are as $[(m_1, m_2, m_3), (n_1, n_2, n_3)]$. A intuitionistic fuzzy multicriteria group

decision-matrix (*IFDM*) is obtained as,

$$IFDM = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \vdots & \vdots & \dots & \vdots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{bmatrix}$$

where $x_{ij} = [(m_{ij}^1, m_{ij}^2, m_{ij}^3), (n_{ij}^1, n_{ij}^2, n_{ij}^3)]$

3. From Intuitionistic fuzzy decision matrix we will do the fuzzification by the method described earlier and get a fuzzy decision matrix as -

$$FDM = \begin{bmatrix} y_{11} & y_{12} & \dots & y_{1n} \\ y_{21} & y_{22} & \dots & y_{2n} \\ \vdots & \vdots & \dots & \vdots \\ y_{m1} & y_{m2} & \dots & y_{mn} \end{bmatrix}$$

where y_{11} takes the shape of $(v_{ij}^1, v_{ij}^2, v_{ij}^3)$, where v_{ij}^k can be evaluated in two ways as

$$v_{ij}^k = \begin{cases} m_{ij}^k + \epsilon_k & \text{if } m_{ij}^k > n_{ij}^k \\ m_{ij}^k & \text{if } m_{ij}^k < n_{ij}^k \\ m_{ij}^k + \frac{\epsilon_k}{2} & \text{if } m_{ij}^k = n_{ij}^k \end{cases}$$

or

$$v_{ij}^k = m_{ij}^k + \epsilon_k \cdot \frac{m_{ij}^k}{m_{ij}^k + n_{ij}^k}$$

where ϵ is hesitancy as $\epsilon_k = 1 - m_{ij}^k - n_{ij}^k$ and $k = 1, 2, 3$.

- then by using the formalization method, Normalized decision matrix is obtained normalization is done as

$$u_{ij}^k = \frac{v_{ij}^k}{\max_{j=1}^m \{v_{ij}^1, v_{ij}^2, v_{ij}^3, \dots\}}$$

- Weighted normalized fuzzy decision matrix is obtained as $\gamma_{ij}^k = w_j^k v_{ij}^k$, $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$ and $k = 1, 2, 3$
- Take Positive ideal solution as (PIS) $A^+ = (1, 1, 1)$ and negative ideal solution (NIS) $A^- = (0, 0, 0)$. Then find the distance from A^+ and A^- for each alternative as D^+ and D^- .
- Find the closeness coefficient for each alternative by formula $C_i = \frac{D_i^-}{D_i^- + D_i^+}$, $i = 1, 2, \dots, m$
- According to closeness coefficient rank the alternatives.

3.1 TOPSIS in fuzzy environment for group decision making

Many extension in TOPSIS are made to handle fuzzy environment, [14,23,46,83]

3.2 Numerical Example

Let there are three experts in order to make the best decision thus, we use fuzzy TOPSIS. Assume that a company requires a locations for their office work. After primary screening three options A_1, A_2, A_3 left . Three decision makers D_1, D_2, D_3 have to decide on the basis of four criterias environment C_1 , safety C_2 , condition C_3 and availability of transport C_4 . Their are four criteria to evaluate the best alternatives. These criteria are C_1 , C_2 , C_3 : and C_4 . Let the fuzzy weights for each criteria are

- The decision-makers use the linguistic weighting variables (shown in Table 1) to assess the importance of the criteria and present it in
- The decision-makers use the linguistic rating variables (shown in Table 2) to evaluate the rating of alternatives with respect to each criterion and present it in
- Converting the linguistic evaluation (shown in Tables) into intuitionistic triangular fuzzy numbers to construct the fuzzy decision matrix and determine the fuzzy weight of each criterion as (0.4, 0.8, 0.5), (1, 0.5, 0.6), (0.4, 1, 0.5) and (1, 1, 0.5) of criteria C_1 , C_2 , C_3 : and C_4 respectively

Triangular Intuitionistic fuzzy decision matrix for three alternatives				
The importance of weights and criterias				
	C_1	C_2	C_3	C_4
A_1	[(0.5, 0.7, 0.9), (0.4, 0.2, 0.1)]	[(0.4, 0.4, 0.3), (0.2, 0.4, 0.5)]	[(0.4, 0.2, 0.4), (0.6, 0.4, 0.3)]	[(0.7, 0.4, 0.3), (0.2, 0.5, 0.6)]
A_2	[(0.2, 0.3, 0.5), (0.5, 0.5, 0.4)]	[(0.4, 1, 0.3), (0.2, 0, 0.6)]	[(0.5, 0.4, 0.4), (0.5, 0.3, 0.5)]	[(0.6, 0.4, 0.7), (0.2, 0.4, 0.2)]
A_3	[(0.3, 0.5, 0.6), (0.5, 0.4, 0.2)]	[(0.4, 0.5, 0.3), (0.2, 0.5, 0.6)]	[(0.7, 0.8, 0.2), (0.2, 0.2, 0.6)]	[(0.3, 0.4, 0.2), (0.7, 0.5, 0.8)]

Table 1

Further we have two methods

3.2.1 Method 1

we will get fuzzy decision matrix for three alternatives by adding hesitancy to higher value if the values are unequal. For unequal values we will add half of hesitancy each to get fuzzy matrix as shown in table 2

Triangular Intuitionistic fuzzy decision matrix for three alternatives and corresponding fuzzy weights				
The importance of weights and criterias				
	C_1	C_2	C_3	C_4
A_1	(0.6, 0.8, 0.9)	(0.8, 0.5, 0.3)	(0.5, 0.75, 0.7)	(0.8, 0.4, 0.3)
A_2	(0.2, 0.3, 0.6)	(0.8, 1, 0.3)	(0.5, 0.7, 0.4)	(0.8, 0.5, 0.8)
A_3	(0.3, 0.6, 0.8)	(0.4, 0.5, 0.3)	(0.8, 0.8, 0.2)	(0.3, 0.4, 0.2)
weights	(0.4, 0.8, 0.5)	(1, 0.5, 0.6)	(0.4, 1, 0.5)	(1, 1, 0.5)

Table 2

Normalized decision matrix is evaluated

The fuzzy normalized decision matrix				
	C_1	C_2	C_3	C_4
A_1	(0.67, 0.89, 1)	(0.8, 0.5, 0.3)	(0.5, 0.75, 0.88)	(1, 0.5, 0.38)
A_2	(0.22, 0.33, 0.67)	(0.8, 1, 0.3)	(0.63, 0.88, 0.5)	(1, 0.63, 1)
A_3	(0.33, 0.67, 0.89)	(0.4, 0.5, 0.3)	(1, 1, 0.25)	(0.38, 0.5, 0.25)
weights	(0.4, 0.8, 0.5)	(1, 0.5, 0.6)	(0.4, 1, 0.5)	(1, 1, 0.5)

Table 3

The fuzzy weighted normalized decision matrix				
	C_1	C_2	C_3	C_4
A_1	(0.27, 0.71, 0.5)	(0.8, 0.25, 0.18)	(0.2, 0.75, 0.44)	(1, 0.5, 0.19)
A_2	(0.88, 0.26, 0.34)	(0.8, 0.5, 0.18)	(0.25, 0.88, 0.25)	(1, 0.63, 0.5)
A_3	(0.13, 0.54, 0.45)	(0.4, 0.25, 0.18)	(0.4, 1, 0.13)	(0.38, 0.5, 0.13)

Table 4

PIS $A^+ = (1, 1, 1)$

NIS $A^- = (0, 0, 0)$

Distance from PIS $A^+ = (1, 1, 1)$

Here we will use the formula for distance between (x_1, x_2, x_3) and (y_1, y_2, y_3)

as $\sqrt{\frac{(y_1-x_1)^2+(y_2-x_2)^2+(y_3-x_3)^2}{3}}$

Distance from PIS of each criteria					Total
	C_1	C_2	C_3	C_4	D_i^+
A_1	0.538	0.652	0.582	0.550	2.322
A_2	0.577	0.664	0.618	0.359	2.218
A_3	0.651	0.729	0.610	0.655	2.671

Table 5

Distance from NIS of each criteria					Total
	C_1	C_2	C_3	C_4	D_i^-
A_1	0.525	0.658	0.515	0.655	2.353
A_2	0.565	0.554	0.548	0.741	2.408
A_3	0.413	0.291	0.626	0.370	2.671

Table 6

relative closeness coefficient is

	D_i^+	D_i^-	$CC_i = \frac{D_i^-}{D_i^+ + D_i^-}$
A_1	2.322	2.353	0.503
A_2	2.218	2.408	0.521
A_3	2.671	1.7	0.389

Table 7

According to closeness coefficient ranking of alternatives is
 A_2, A_1, A_3
 The best candidate is A_2 .

3.3 Method 2

In this we will divide the hesitancy in proportion and from table 1 we have,

The matrix fuzzified				
	C_1	C_2	C_3	C_4
A_1	(0.56, 0.78, 0.9)	(0.67, 0.5, 0.38)	(0.4, 0.33, 0.57)	(0.78, 0.44, 0.33)
A_2	(0.29, 0.38, 0.56)	(0.67, 1, 0.33)	(0.5, 0.57, 0.44)	(0.75, 0.5, 0.78)
A_3	(0.38, 0.56, 0.75)	(0.67, 0.5, 0.33)	(0.78, 0.8, 0.25)	(0.3, 0.44, 0.2)
weights	(0.4, 0.8, 0.5)	(1, 0.5, 0.6)	(0.4, 1, 0.5)	(1, 1, 0.5)

Table 8

Normalized decision matrix is evaluated

The fuzzy normalized decision matrix				
	C_1	C_2	C_3	C_4
A_1	(0.62, 0.87, 1)	(0.67, 0.5, 0.38)	(0.2, 0.41, 0.71)	(1, 0.56, 0.42)
A_2	(0.32, 0.42, 0.62)	(0.67, 1, 0.33)	(0.23, 0.71, 0.73)	(0.96, 0.64, 1)
A_3	(0.42, 0.62, 0.83)	(0.67, 0.5, 0.33)	(0.98, 1, 0.31)	(0.38, 0.56, 0.26)
weights	(0.4, 0.8, 0.5)	(1, 0.5, 0.6)	(0.4, 1, 0.5)	(1, 1, 0.5)

Table 9

The fuzzy weighted normalized decision matrix				
	C_1	C_2	C_3	C_4
A_1	(0.25, 0.70, 0.5)	(0.67, 0.25, 0.23)	(0.08, 0.41, 0.36)	(1, 0.5, 0.21)
A_2	(0.13, 0.34, 0.31)	(0.67, 0.5, 0.20)	(0.09, 0.71, 0.37)	(0.96, 0.64, 0.5)
A_3	(0.17, 0.50, 0.42)	(0.67, 0.25, 0.20)	(0.4, 1, 0.16)	(0.38, 0.56, 0.13)

Table 10

PIS $A^+ = (1, 1, 1)$

NIS $A^- = (0, 0, 0)$

Distance from PIS $A^+ = (1, 1, 1)$

Here we will use the formula for distance between (x_1, x_2, x_3) and (y_1, y_2, y_3)
 as $\sqrt{\frac{(y_1-x_1)^2+(y_2-x_2)^2+(y_3-x_3)^2}{3}}$

Distance from PIS of each criteria					Total
	C_1	C_2	C_3	C_4	D_i^+
A_1	0.548	0.649	0.731	0.540	2.468
A_2	0.746	0.577	0.661	0.356	2.34
A_3	0.652	0.661	0.661	0.667	2.641

Table 11

Distance from NIS of each criteria					Total
	C_1	C_2	C_3	C_4	D_i^-
A_1	0.517	0.438	0.318	0.657	1.93
A_2	0.276	0.438	0.465	0.726	1.905
A_3	0.390	0.429	0.465	0.398	1.682

Table 12

relative closeness coefficient is

	D_i^+	D_i^-	$CC_i = \frac{D_i^-}{D_i^+ + D_i^-}$
A_1	2.468	1.93	$\frac{1.93}{1.93+2.468} = 0.439$
A_2	2.34	1.905	$\frac{1.905}{2.34+1.905} = 0.449$
A_3	2.641	1.682	$\frac{1.682}{2.641+1.682} = 0.389$

Table 13

According to closeness coefficient ranking of alternatives is

A_2, A_1, A_3

The best candidate is A_2

4 Conclusions

In this paper we have constructed a technique to find a solution using topsis for the in triangular intuitionistic fuzzy numbers. This technique give idea to find a solution to deal with trapezoidal intuitionistic or octagonal intuitionistic fuzzy numbers. For future research we can use different MCDM techniques for triangular, trapezoidal, octagonal or others intuitionistic fuzzy numbers.

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